Compilers

Intermediate Representations and Data-Flow Analysis

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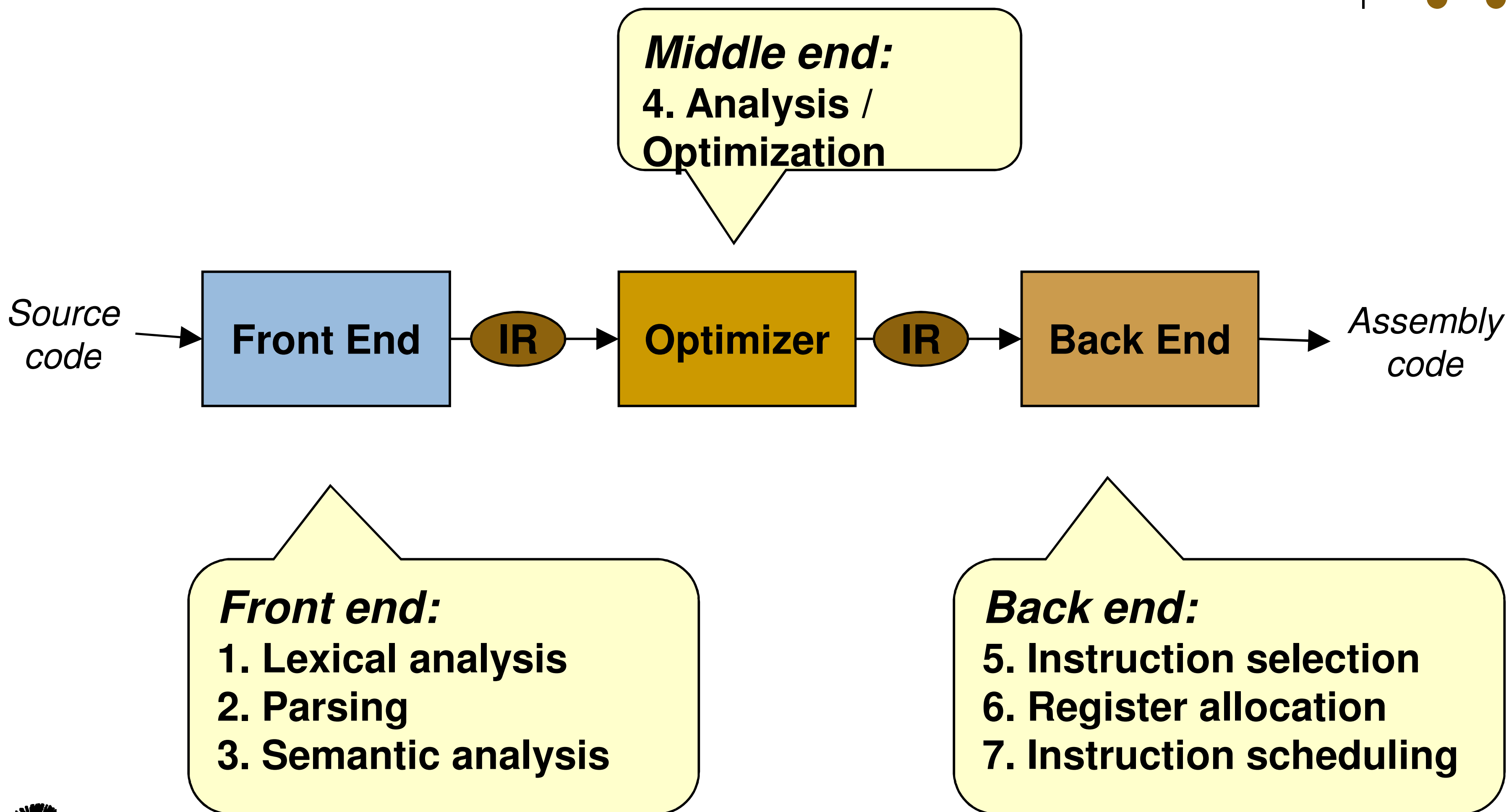
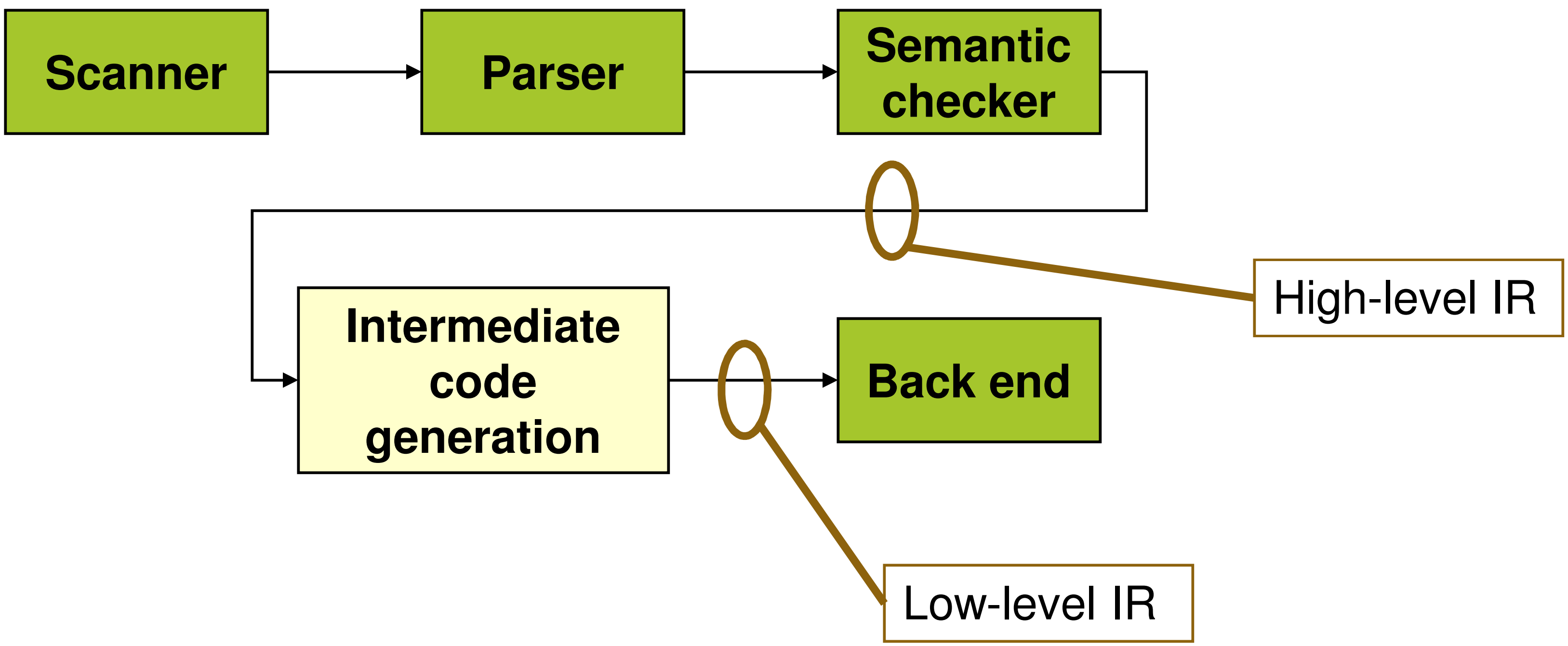
(original slides by Sam Guyer@Tufts)

Modern optimizing compiler



A bit more detail

- Intermediate representations and code generation



Low-level IR

- Linear stream of ***abstract instructions***
- Instruction: single operation and assignment

```
x = y op z
```

```
x ← y op z
```

```
op x, y, z
```

- Must break down high-level constructs
  - Example:

```
z = x - 2 * y
```

```
t ← 2 * y
z ← x - t
```

  - Introduce temps as necessary: called ***virtual registers***
- Simple control-flow
  - Label and goto

```
label1:
goto label1
if_goto x, label1
```

Jump to label1 if x has non-zero value

Stack machines

Now, Java VM

- Originally for stack-based computers

```
x – 2 * y
```

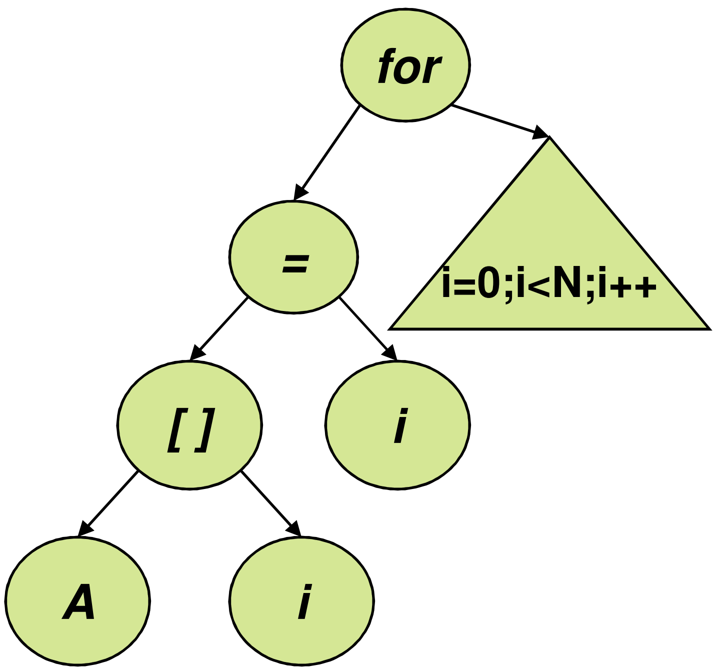
```
push x
push 2
push y
multiply
subtract
```

Post-fix notation

- What are advantages?
  - Introduced names are *implicit*, not *explicit*
  - Simple to generate and execute code
  - Compact form – who cares about code size?
    - Embedded systems
    - Systems where code is transmitted (the 'Net)

IR Trade-offs

```
for (i=0; i<N; i++)
  A[i] = i;
```




```
loop:
temp1 = &A
temp2 = i * 4
temp3 = temp1 + temp2
store [temp3] = i
...
goto loop
```

Loop invariant

Strength reduce to temp2 += 4

Towards code generation

```
if (c == 0) {
  while (c < 20) {
    c = c + 2;
  }
}
else
  c = n * n + 2;
```

```
t1 = c == 0
if_goto t1, lab1
t2 = n * n
c = t2 + 2
goto end
lab1:
t3 = c >= 20
if_goto t3, end
c = c + 2
goto lab1
end:
```

Motivating Example: Dead code elimination

- **Idea**:
  - Remove a computation if result is never used

```
y = w - 7;
x = y + 1;
y = 1;
x = 2 * z;
```

→

```
y = w - 7;

y = 1;
x = 2 * z;
```

→

```


y = 1;
x = 2 * z;
```

- Safety
  - Variable is dead if it is never used after defined
  - Remove code that assigns to dead variables
- This may, in turn, create more dead code
  - Dead-code elimination usually works transitively

Dead code

- Another example:

```
x = y + 1;
y = 2 * z;
x = y + z;
z = 1;
z = x;
```

Which statements can be safely removed?

- Conditions:
  - Computations whose value is never used
  - Obvious for straight-line code
  - What about control flow?

Dead code

- With if-then-else:

Which statements are can be removed?

```
x = y + 1;
y = 2 * z;
if (c) x = y + z;
z = 1;
z = x;
```

- Which statements are dead code?
  - What if “c” is false?
  - Dead only on some paths through the code

Dead code

- And a loop:

Which statements are can be removed?

```
while (p) {
  x = y + 1;
  y = 2 * z;
  if (c) x = y + z;
  z = 1;
}
z = x;
```

- Now which statements are dead code?

Dead code

- And a loop:

Which statements are can be removed?

```
while (p) {
  x = y + 1;
  y = 2 * z;
  if (c) x = y + z;
  z = 1;
}
z = x;
```

- Statement “x = y+1” not dead
- What about “z = 1”?

Low-level IR

- Most optimizations performed in low-level IR
  - Labels and jumps
  - No explicit loops
- Even harder to see possible paths

```
label1:
jumpifnot p label2
x = y + 1
y = 2 * z
jumpifnot c label3
x = y + z
label3:
z = 1
jump label1
label2:
z = x
```

Optimizations and control flow

- Dead code is ***flow sensitive***
  - Not obvious from program

    Dead code example: are there any possible paths that make use of the value?
  - Must characterize all possible dynamic behavior
  - Must verify conditions at compile-time
- Control flow makes it hard to extract information
  - High-level: different kinds of control structures
  - Low-level: control-flow hard to infer
- Need a unifying data structure

Control flow graph

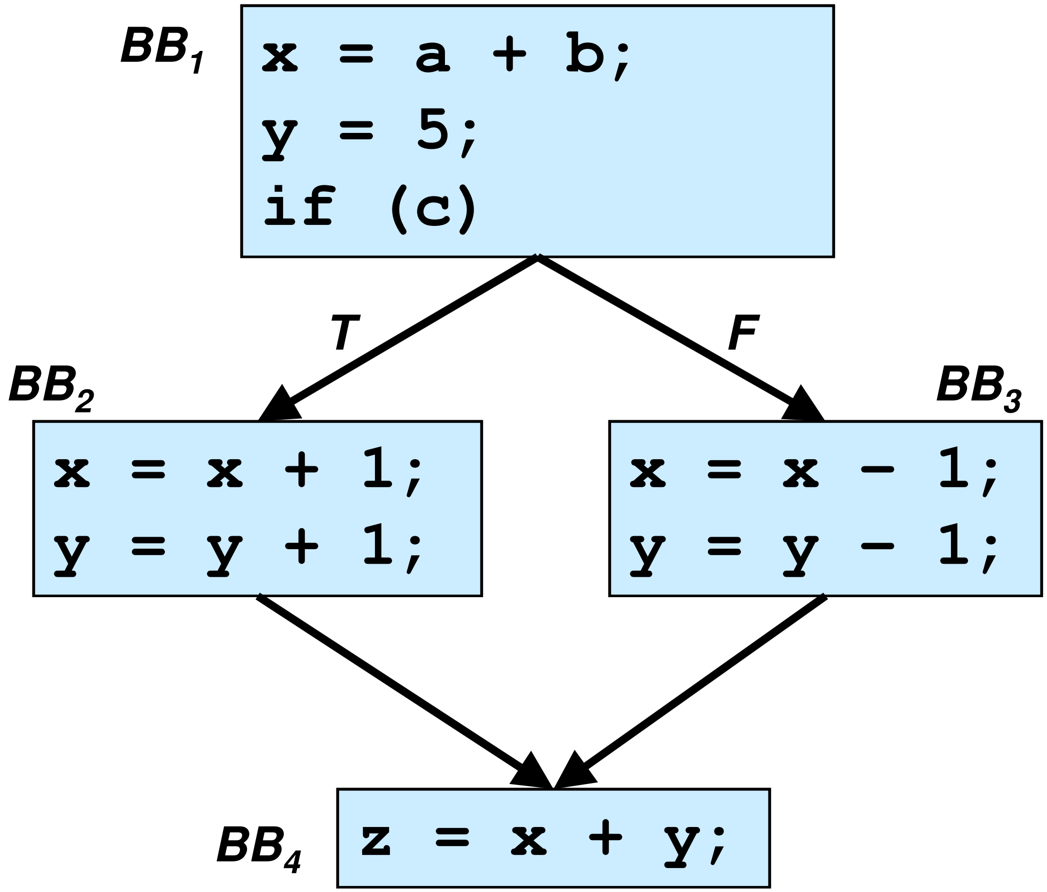
- ***Control flow graph*** (CFG):
  a graph representation of the program
  - Includes both computation and control flow
  - Easy to check control flow properties
  - Provides a framework for global optimizations and other compiler passes
- Nodes are ***basic blocks***
  - Consecutive sequences of non-branching statements
- Edges represent control flow
  - From jump to a label
  - Each block may have multiple incoming/outgoing edges

CFG Example

Program

```
x = a + b;
y = 5;
if (c) {
  x = x + 1;
  y = y + 1;
} else {
  x = x - 1;
  y = y - 1;
}
z = x + y;
```

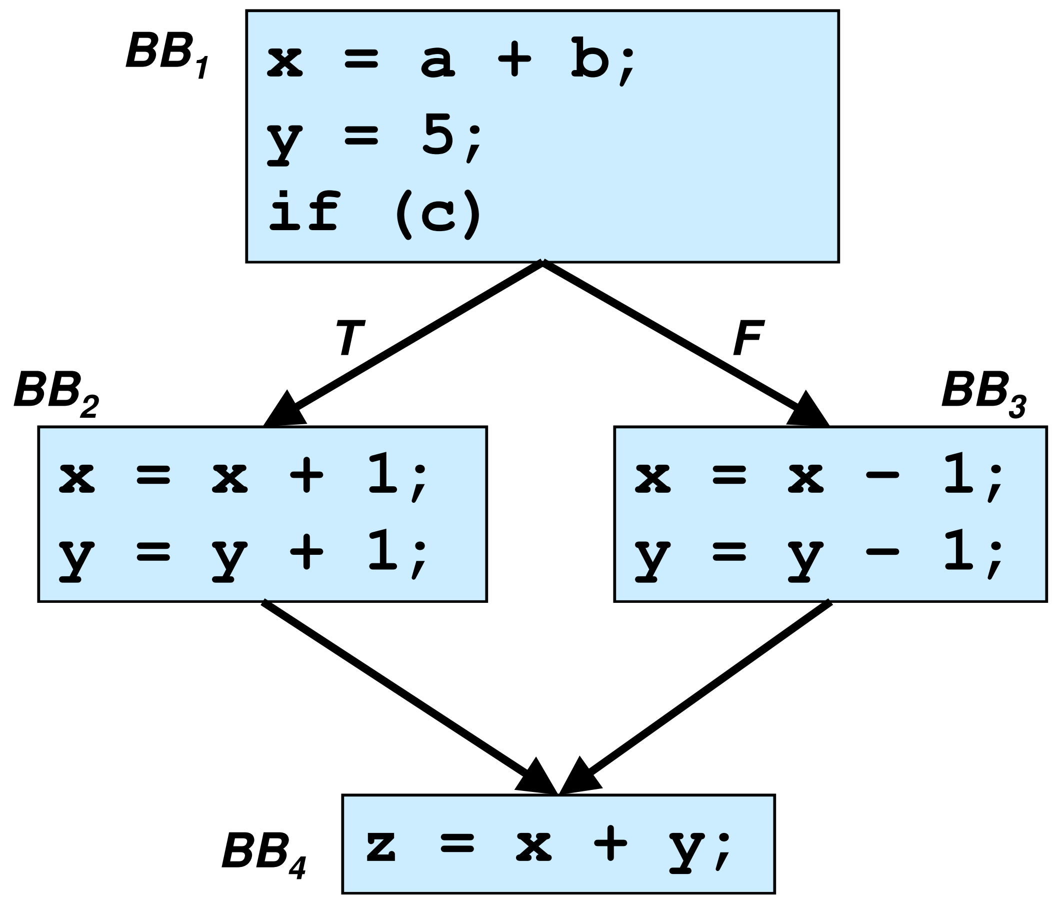
Control flow graph



Multiple program executions

- CFG models all program executions
- An actual execution is a path through the graph
- Multiple paths: multiple possible executions
  - How many?

Control flow graph

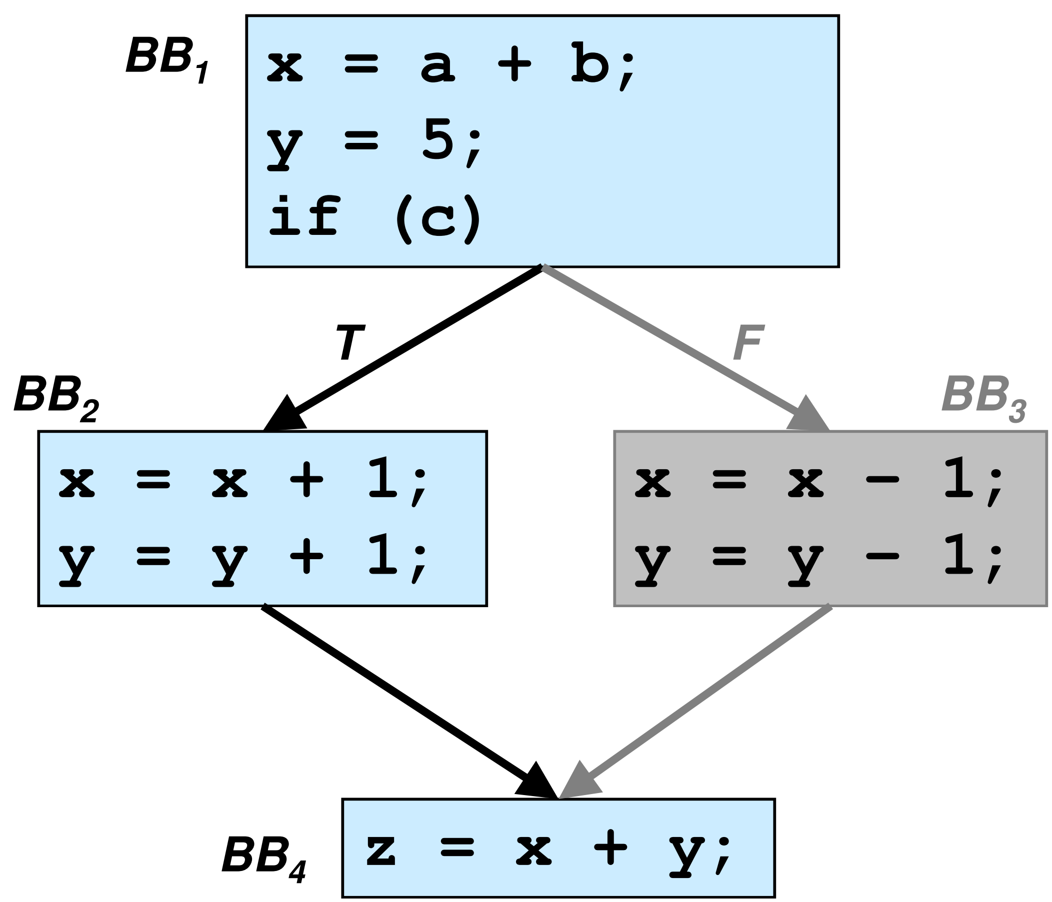


Execution 1

- CFG models all program executions

- Execution 1:
  - c is true
  - Program executes BB_1, BB_2, and BB_4

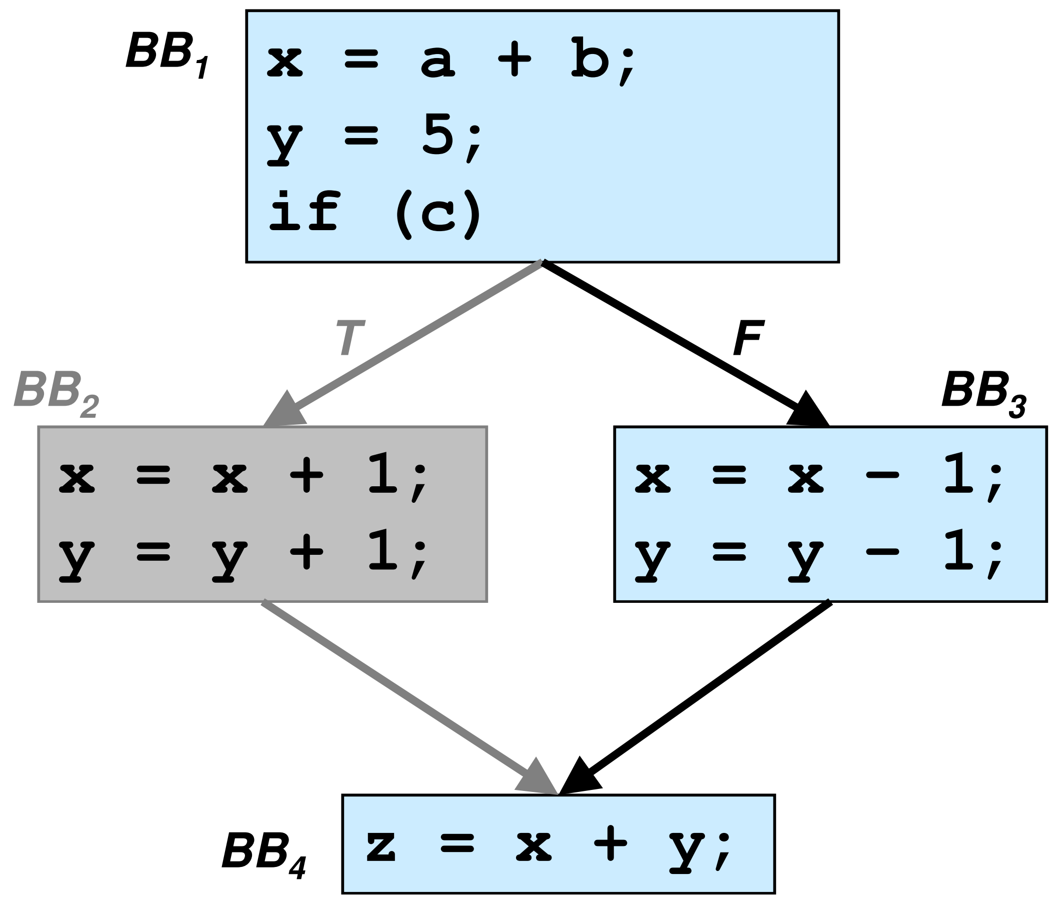
Control flow graph



Execution 2

- CFG models all program executions
- Execution 2:
  - c is false
  - Program executes BB_1, BB_3, and BB_4

Control flow graph



Basic blocks

- **Idea**:
  - Once execution enters the sequence, all statements (or instructions) are executed
  - Single-entry, single-exit region
- Details
  - Starts with a label
  - Ends with one or more branches
  - Edges may be labeled with predicates

    May include special categories of edges
    - Exception jumps
    - Fall-through edges
    - Computed jumps (jump tables)

Building the CFG

- Two passes
  - First, group instructions into basic blocks
  - Second, analyze jumps and labels
- How to identify basic blocks?
  - Non-branching instructions

    Control cannot flow out of a basic block without a jump
  - Non-label instruction

    Control cannot enter the middle of a block without a label

Basic blocks

- Basic block starts:
  - At a label
  - After a jump
- Basic block ends:
  - At a jump
  - Before a label

```
label1:
jumpifnot p label2
x = y + 1
y = 2 * z
jumpifnot c label3
x = y + z
label3:
z = 1
jump label1
label2:
z = x
```

Basic blocks

- Basic block starts:
  - At a label
  - After a jump
- Basic block ends:
  - At a jump
  - Before a label
- **Note**: order still matters

```
label1:
jumpifnot p label2
```

```
x = y + 1
y = 2 * z
jumpifnot c label3
```

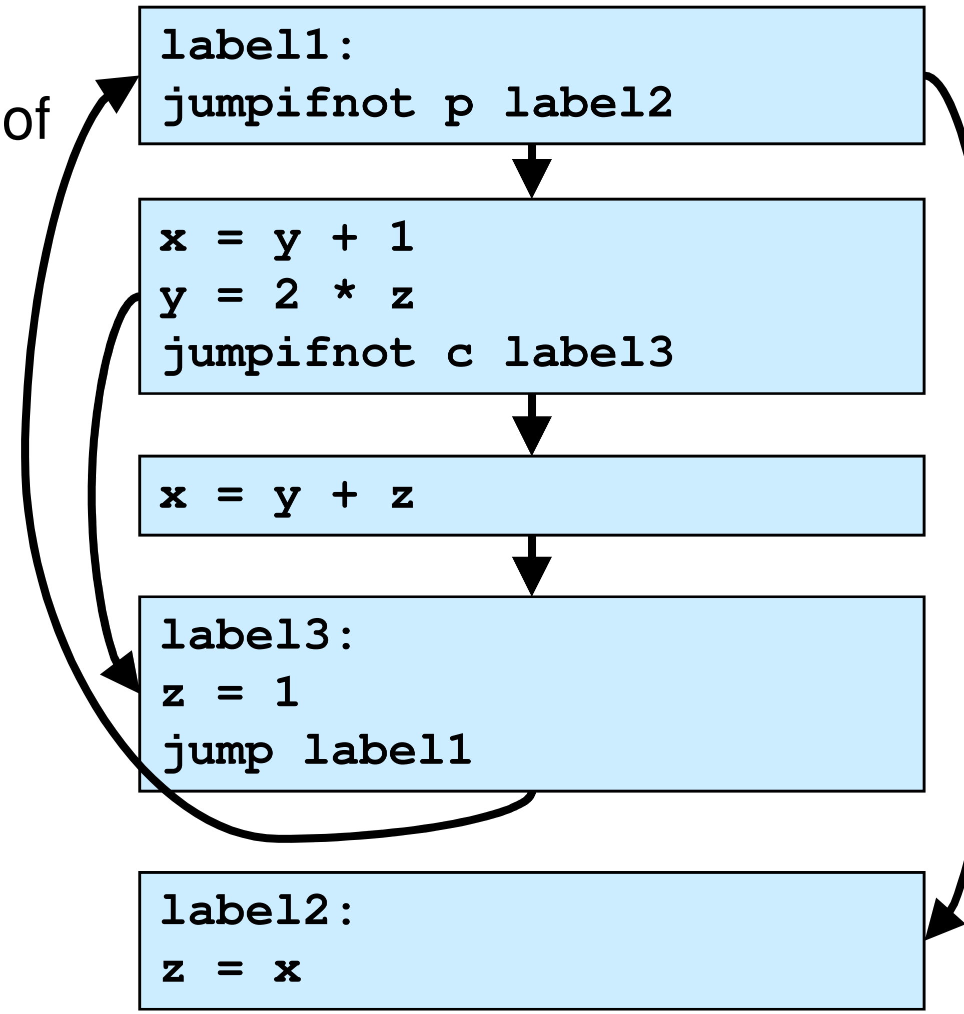
```
x = y + z
```

```
label3:
z = 1
jump label1
```

```
label2:
z = x
```

Add edges

- Unconditional jump
  - Add edge from source of jump to the block containing the label
- Conditional jump
  - 2 successors
  - One may be the fall-through block
- Fall-through



Two CFGs

- From the high-level
  - Break down the complex constructs
  - Stop at sequences of non-control-flow statements
  - Requires special handling of break, continue, goto
- From the low-level
  - Start with lowered IR – tuples, or 3-address ops
  - Build up the graph
  - More general algorithm
  - Most compilers use this approach
- Should lead to roughly the same graph

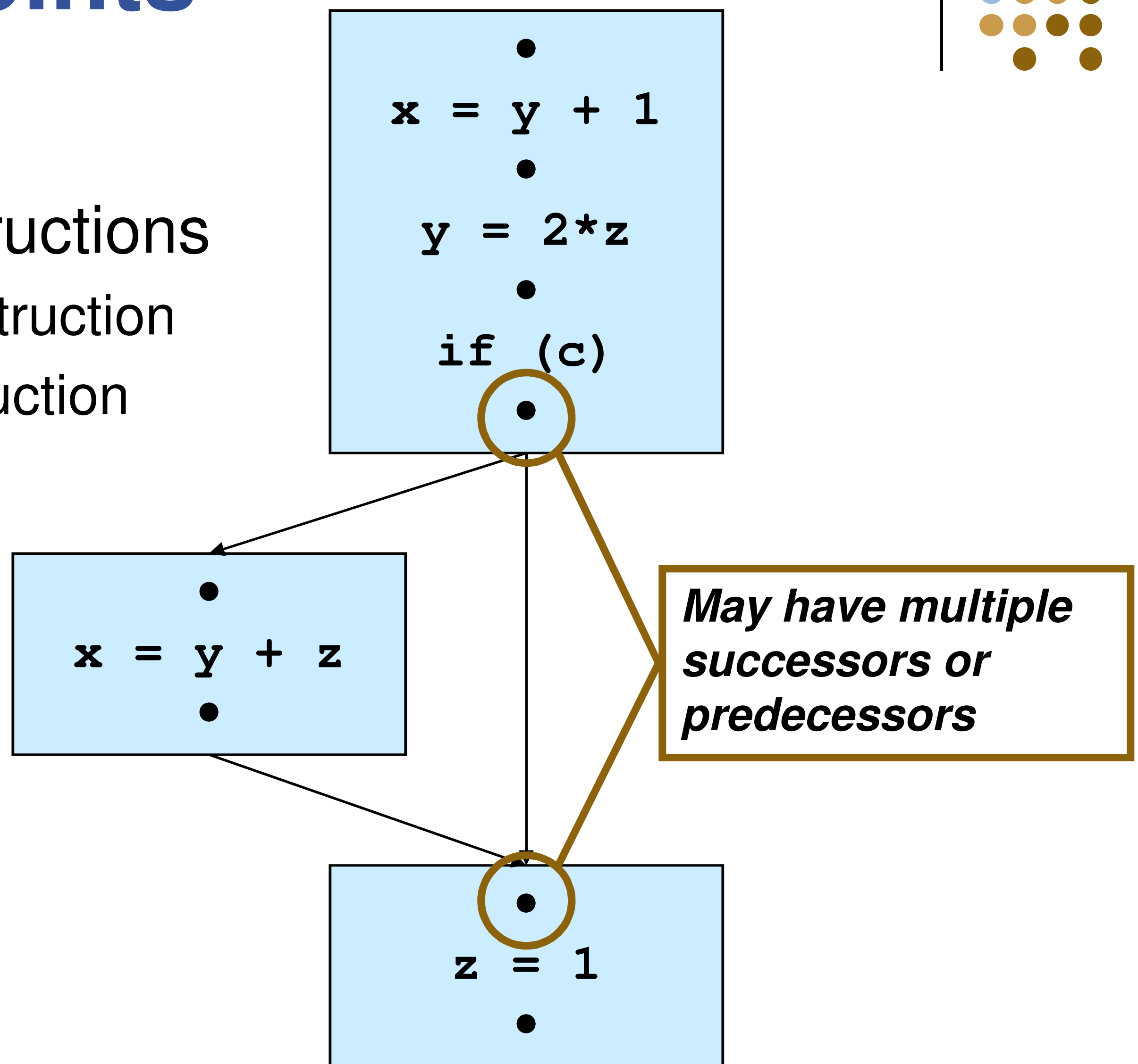
Using the CFG

- Uniform representation for program behavior
  - Shows all possible program behavior
  - Each execution represented as a path
  - Can reason about potential behavior
    Which paths can happen, which can't
  - Possible paths imply possible values of variables

- Example: ***liveness*** information
- **Idea**:
  - Define program points in CFG
  - Describe how information flows between points

Program points

- In between instructions
  - Before each instruction
  - After each instruction



Live variables analysis

- **Idea**
  - Determine ***live range*** of a variable

    Region of the code between when the variable is assigned and when its value is used
  - Specifically:

    Def: A variable v is live at point p if
    - There is a path through the CFG from p to a use of v
    - There are no assignments to v along the path

  ⇒ Compute a set of live variables at each point p

- Uses of live variables:
  - Dead-code elimination – find unused computations
  - Also: register allocation, garbage collection

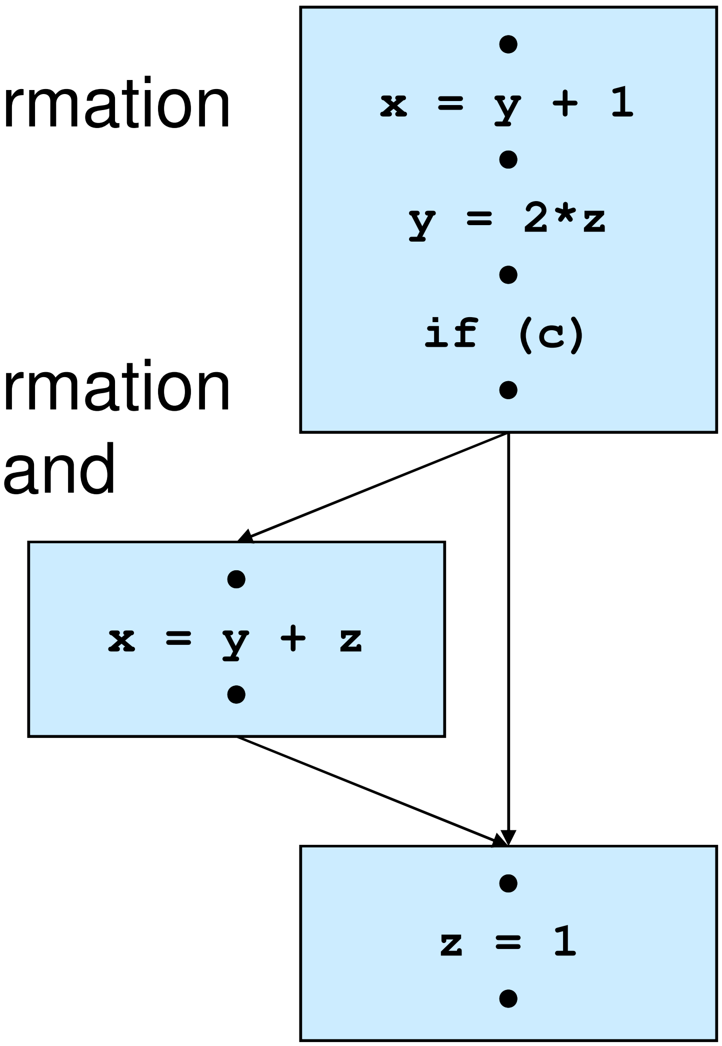
Computing live variables

- How do we compute live variables?
  (Specifically, a set of live variables at each program point)
- What is a straight-forward algorithm?
  - Start at uses of v, search backward through the CFG
  - Add v to live variable set for each point visited
  - Stop when we hit assignment to v
- Can we do better?
  - Can we compute liveness for all variables at the same time?
  - **Idea**:
    - Maintain a set of live variables
    - Push set through the CFG, updating it at each instruction

Flow of information

- ***Question 1***: how does information flow across instructions?

- ***Question 2***: how does information flow between predecessor and successor blocks?



Live variables analysis

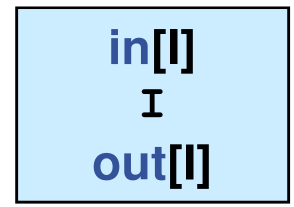
- At each program point:

  Which variables contain values computed earlier and needed later

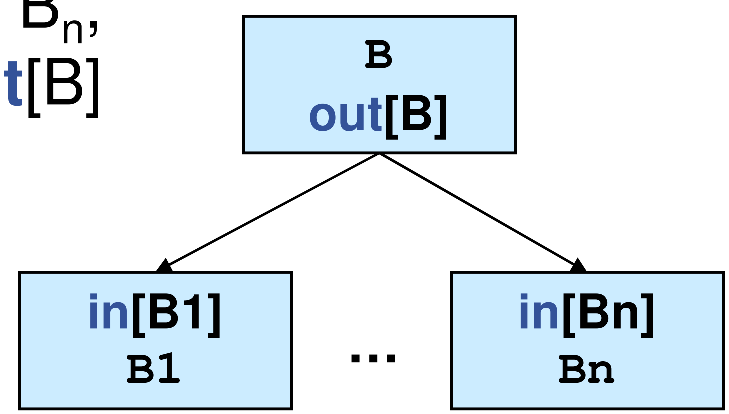
- For instruction I:
  - **in**[I]   : live variables at program point before I
  - **out**[I] : live variables at program point after I
- For a basic block B:
  - **in**[B]   : live variables at beginning of B
  - **out**[B] : live variables at end of B
- **Note**: **in**[I] = **in**[B] for first instruction of B
  out[I] = **out**[B] for last instruction of B

Computing liveness

- ***Answer question 1***: for each instruction I, what is relation between **in**[I] and **out**[I]?



- ***Answer question 2***: for each basic block B, with successors B_1, …, B_n, what is relationship between **out**[B] and **in**[B_1] … **in**[B_n]



Part 1: Analyze instructions

- Live variables across instructions
- Examples:

in[I] = {y,z}	in[I] = {y,z,t}	in[I] = {x,t}
`x = y + z`	`x = y + z`	`x = x + 1`
out[I] = {x}	out[I] = {x,t,y}	out[I] = {x,t}

- Is there a general rule?

Liveness across instructions

- How is liveness determined?
  - All variables that I uses are live before I

    *Called the **uses** of* I
  - All variables live after I are also live before I, unless I writes to them

    *Called the **defs** of* I
- Mathematically:

in[I] = {b}

```
a = b + 2
```

in[I] = {y,z}

```
x = 5
```

out[I] = {x,y,z}

$$\textbf{in}[I] = (\ \textbf{out}[I] - \textbf{def}[I]\ ) \cup \textbf{use}[I]$$

Example

- Single basic block (obviously: **out**[I] = **in**[succ(I)])
  - Live1 = in[B] = in[I1]
  - Live2 = out[I1] = in[I2]
  - Live3 = out[I2] = in[I3]
  - Live4 = out[I3] = out[B]
- Relation between live sets
  - Live1 = (Live2 – {x}) ∪ {y}
  - Live2 = (Live3 – {y}) ∪ {z}
  - Live3 = (Live4 – {}) ∪ {d}

Live1

```
x = y+1
```

Live2

```
y = 2*z
```

Live3

```
if (d)
```

Live4

Flow of information

- Equation:

$$in[l] = (out[l] - def[l]) \cup use[l]$$

- Notice: information flows ***backwards***
  - Need out[] sets to compute in[] sets
  - Propagate information up
- Many problems are ***forward***

  Common sub-expressions, constant propagation, others

Live1

```
x = y+1
```

Live2

```
y = 2*z
```

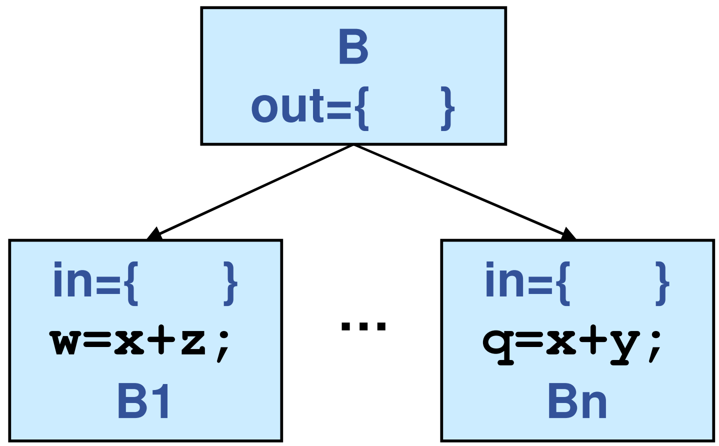
Live3

```
if (d)
```

Live4

Part 2: Analyze control flow

- ***Question 2***: for each basic block B, with successors B_1, …, B_n, what is relationship between **out**[B] and **in**[B_1] … **in**[B_n]
- Example:



- What’s the general rule?

Control flow

- Rule: A variable is live at end of block B if it is live at the beginning of ***any*** of the successors
  - Characterizes all possible executions
  - ***Conservative***: some paths may not actually happen

- Mathematically:

$$\textbf{out}[B] = \bigcup_{B' \in succ(B)} \textbf{in}[B']$$

- Again: information flows backwards

System of equations

- Put parts together:

$$\textbf{in}[I] = (\textbf{out}[I] - \textbf{def}[I]) \cup \textbf{use}[I]$$
$$\textbf{out}[I] = \textbf{in}[succ(I)]$$
$$\textbf{out}[B] = \bigcup_{B' \in succ(B)} \textbf{in}[B']$$

Often called a system of ***Dataflow Equations***

- Defines a system of equations (or constraints)
  - Consider equation instances for each instruction and each basic block
  - What happens with loops?
    - Circular dependences in the constraints
    - Is that a problem?

Solving the problem

- Iterative solution:
  - Start with empty sets of live variables
  - Iteratively apply constraints
  - Stop when we reach a ***fixpoint***

For all instructions in[I] = out[I] = ∅

Repeat

For each instruction I

in[I] = (out[I] – def[I]) ∪ use[I]

out[I] = in[succ(I)]

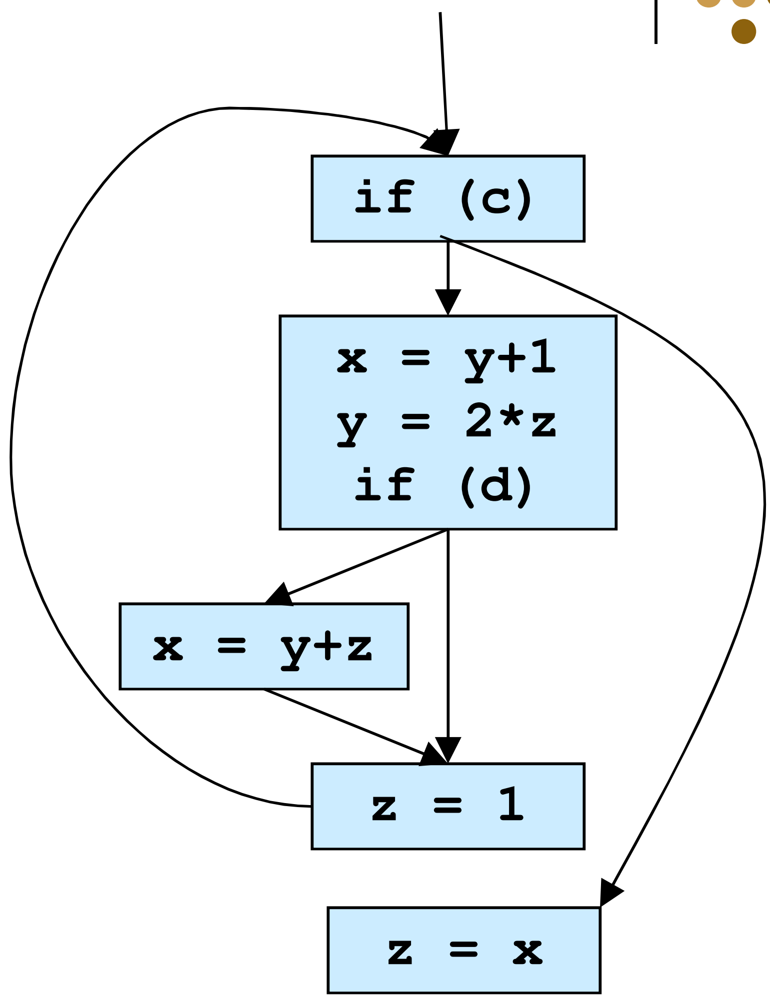
For each basic block B

$$out[B] = \bigcup_{B' \in succ(B)} in[B']$$

Until no new changes in sets

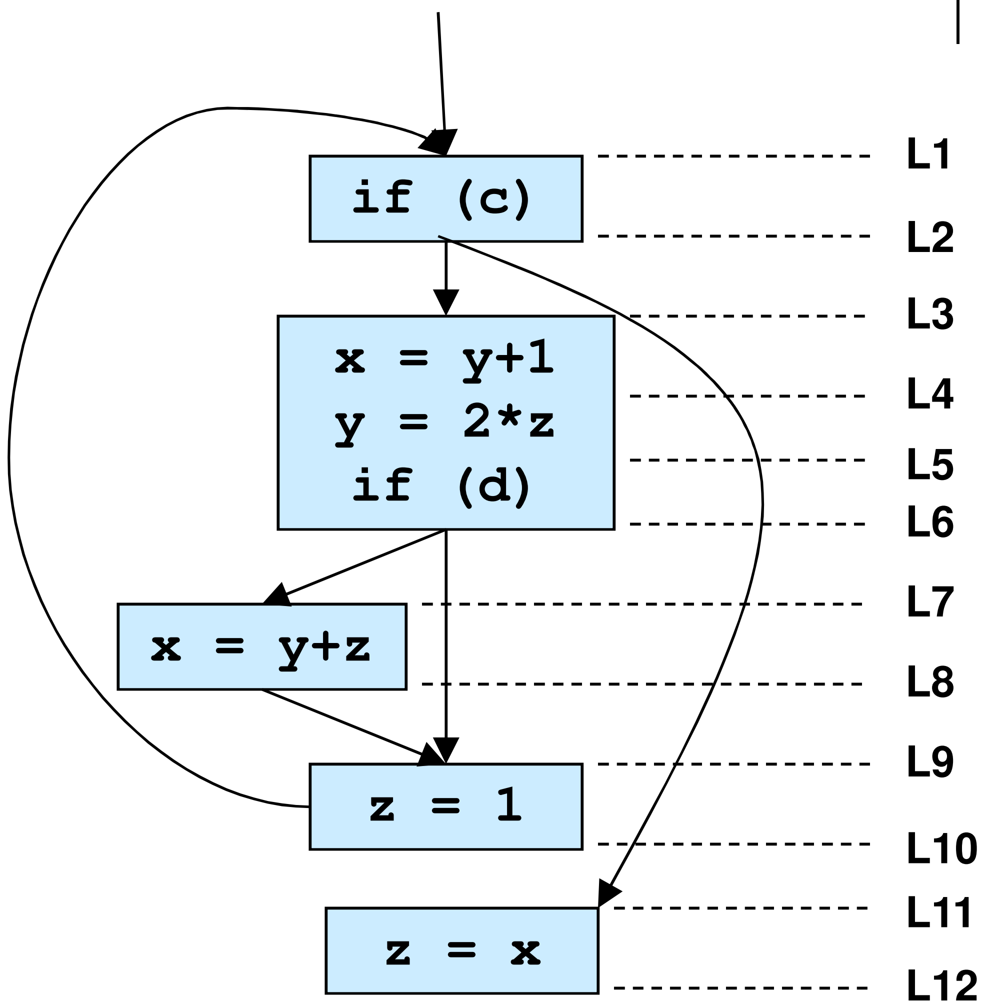
Example

- Steps:
  - Set up live sets for each program point
  - Instantiate equations
  - Solve equations



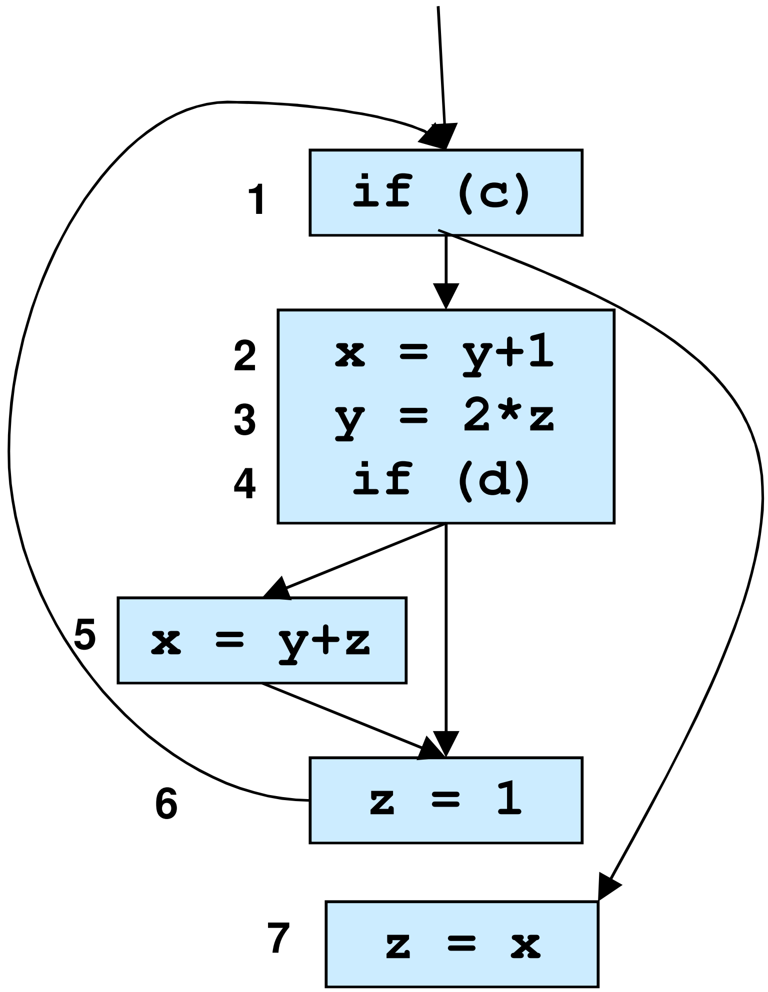
Example

- Program points



Example

L1 = L2 ∪ {c}
L2 = L3 ∪ L11
L3 = (L4 – {x}) ∪ {y}
L4 = (L5 – {y}) ∪ {z}
L5 = L6 ∪ {d}
L6 = L7 ∪ L9
L7 = (L8 – {x}) ∪ {y,z}
L8 = L9
L9 = L10 – {z}
L10 = L1
L11 = (L12 – {z}) ∪ {x}
L12 = {}



L1 = { x, y, z, c, d }
L2 = { x, y, z, c, d }
L3 = { y, z, c, d }
L4 = { x, z, c, d }
L5 = { x, y, z, c, d }
L6 = { x, y, z, c, d }
L7 = { y, z, c, d }
L8 = { x, y, c, d }
L9 = { x, y, c, d }
L10 = { x, y, z, c, d }
L11 = { x }
L12 = { }

Questions

- Does this terminate?
- Does this compute the right answer?
- How could generalize this scheme for other kinds of analysis?

Generalization

- Dataflow analysis
  - A common framework for such analysis
  - Computes information at each program point
  - Conservative: characterizes all possible program behaviors

- Methodology
  - Describe the information (e.g., live variable sets) using a structure called a ***lattice***
  - Build a system of equations based on:
    - How each statement affects information
    - How information flows between basic blocks
  - Solve the system of constraints

Parts of live variables analysis

- Live variable sets
  - Called ***flow values***
  - Associated with program points
  - Start "empty", eventually contain solution
- Effects of instructions
  - Called ***transfer functions***
  - Take a flow value, compute a new flow value that captures the effects
  - One for each instruction – often a schema
- Handling control flow
  - Called ***confluence operator***
  - Combines flow values from different paths

Mathematical model

- Flow values
  - Elements of a lattice $L = (P, \subseteq)$
  - Flow value $v \in P$
- Transfer functions
  - Set of functions (one for each instruction)
  - $F_i : P \rightarrow P$
- Confluence operator
  - Merges lattice values
  - $C : P \times P \rightarrow P$
- How does this help us?

Lattices

- Lattice L = (P, $\sqsubseteq$)
- A partial order relation $\sqsubseteq$

  Reflexive, anti-symmetric, transitive
- Upper and lower bounds

  Consider a subset S of P
  - Upper bound of S: $u \in S : \forall x \in S \;\; x \sqsubseteq u$
  - Lower bound of S: $l \in S : \forall x \in S \;\; l \sqsubseteq x$
- Lattices are complete

  Unique greatest and least elements
  - "Top" $T \in P : \forall x \in P \;\; x \sqsubseteq T$
  - "Bottom" $\bot \in P : \forall x \in P \; \bot \sqsubseteq x$

Confluence operator

- Combine flow values
  - "Merge" values on different control-flow paths
  - Result should be a safe over-approximation
  - We use the lattice $\sqsubseteq$ to denote "more safe"

- Example: live variables
  - v1 = {x, y, z} and v2 = {y, w}
  - How do we combine these values?
  - $v = v1 \cup v2 = \{w, x, y, z\}$
  - What is the "$\sqsubseteq$" operator?
  - Superset

Meet and join

- **Goal**:
  Combine two values to produce the "best" approximation
  - Intuition:
    - Given v1 = {x, y, z} and v2 = {y, w}
    - A safe over-approximation is "all variables live"
    - We want the smallest set
- Greatest lower bound
  - Given $x,y \in P$
  - $GLB(x,y) = z$ such that
    - $z \subseteq x$ and $z \subseteq y$ and
    - $\forall w \; w \subseteq x$ and $w \subseteq y \Rightarrow w \subseteq z$
  - ***Meet*** operator: $x \wedge y = GLB(x, y)$
- Natural "opposite": Least upper bound, ***join*** operator

Termination

- Monotonicity

  Transfer functions F are ***monotonic*** if

  - Given $x,y \in P$
  - If $x \subseteq y$ then $F(x) \subseteq F(y)$
  - Alternatively: $F(x) \subseteq x$

- Key idea:

  Iterative dataflow analysis terminates if

  - Transfer functions are monotonic
  - Lattice has finite height
  - *Intuition*: values only go down, can only go to bottom

Example

- Prove monotonicity of live variables analysis
  - Equation: $\mathbf{in}[i] = (\ \mathbf{out}[i] - \mathbf{def}[i]\ ) \cup \mathbf{use}[i]$
    (For each instruction i)

  - As a function: $F(x) = (x - def[i]) \cup use[i]$
  - Obligation: If $x \subseteq y$ then $F(x) \subseteq F(y)$
  - Prove:

    $x \subseteq y \quad \Rightarrow \quad (x - def[i]) \cup use[i] \subseteq (y - def[i]) \cup use[i]$

    - Somewhat trivially:
    - $x \subseteq y \Rightarrow x - s \subseteq y - s$
    - $x \subseteq y \Rightarrow x \cup s \subseteq y \cup s$

Dataflow solution

- Question:
  - What is the solution we compute?
  - Start at lattice top, move down
  - Called greatest ***fixpoint***
  - Where does approximation come from?
  - Confluence of control-flow paths
- Ideal solution?
  - Consider each path to a program point separately
  - Combine values at end
  - Called ***meet-over-all-paths*** solution (MOP)
  - When is the fixpoint equal to MOP?

Dataflow solution

- Question:
  - What is the solution we compute?
  - Start at lattice top, move down
  - Called greatest ***fixpoint***
  - Where does approximation come from?
  - Confluence of control-flow paths
- Knaster Tarski theorem
  - Every monotonic function F over a complete lattice L has a unique least (and greatest) fixpoint
  - (Actually, the theorem is more general)

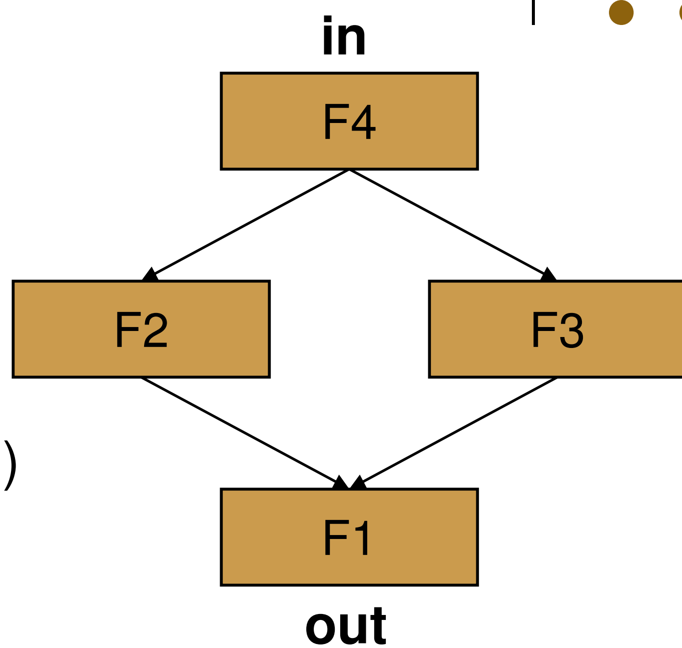
Composition of functions

Consider if-then-else graph

- If we compute each path:
  - in = F4(F2(F1(out)))
  - in = F4(F3(F1(out)))
- Two solutions

  MOP:
  - in = F4(F2(F1(out))) ∧ F4(F3(F1(out)))

  Fixpoint:
  - Merge live vars before applying F4
  - in = F4(F2(F1(out)) ∧ F3(F1(out)))
- When are these two results the same?
  - When the transfer functions are ***distributive***
  - Prove: F(x) ∧ F(y) = F(x ∧ y)



Summary

- Dataflow analysis
  - Lattice of flow values
  - Transfer functions (encode program behavior)
  - Iterative fixpoint computation

- **Key insight**:

  If our dataflow equations have these properties:
  - Transfer functions are monotonic
  - Lattice has finite height
  - Transfer functions distribute over meet operator

  Then:
  - Our fixpoint computation will terminate
  - Will compute meet-over-all-paths solution